



Five Myths of Mastery in Mathematics

Introduction

This document is intended to support NAMA members in their work with schools. It came about as a result of internal discussions within NAMA over the period May to December 2015. At a NAMA professional development session in May 2015 participants noted that many schools are now using some form of mastery in mathematics, or are planning to do so. They also noted that there were sometimes misunderstandings of what mastery approaches to teaching and learning might involve. We identified five specific false ideas that are circulating about mastery in mathematics. We have called these the Five Myths of Mastery in Mathematics.

1. Mastery in mathematics has a single clear definition

Mastery learning is not a new idea. Its origins can be traced to the early work of Benjamin Bloom (Bloom, 1971a, Bloom, 1968, Bloom, 1971b). In 1990 a meta-analysis of 108 mainly USA studies of the effectiveness of mastery learning concluded that it raises achievement (Kulik et al., 1990). The effects appeared to be greater for lower achieving students and they depended on a number of factors, including which mastery procedures were used. This immediately tells us that there were different mastery approaches being used in the 1980s to operationalise Bloom's ideas.

Here in England, the NCETM and MathsHubs are now promoting mastery approaches to mathematics drawing on practices in east and south-east Asia. Separately from this, there is an organisation, Mathematics Mastery, linked to the Ark Academy chain, which takes a slightly different approach to mastery in mathematics. NCETM and MathsHubs, and Mathematics Mastery provide extensive web-based advice on mastery in mathematics (see later references to some of their publications). In October 2015 NRICH posted views on Mastering Mathematics and Problem Solving (NRICH, 2015). Although there are some differences in these approaches to mastery, the major thrusts are consistent.

Teachers are currently seeking to make sense of mastery in mathematics, some in the belief that there is an agreed definition being used within the English school system and that this definition is based on a meaning derived from maths teaching and learning in some high performing jurisdictions, in particular Shanghai and Singapore. As indicated the existence of a single definition is a myth. The ideas around mastery, have been interpreted and developed in different ways. Furthermore the term 'mastery' is being used in conjunction with different aspects of education including mastery curriculum, mastery teaching, mastery goals and mastery level in assessment. This has led to a range of conceptualisations.

Mastery goal orientation is defined by Carol Dweck (Dweck, 1986) as one where learners seek to develop their competence by acquiring new skills and mastering new situations, with a focus on personal improvement and development. This is contrasted with a performance goal orientation where learners seek to demonstrate and prove their ability to others in order to receive favourable judgments and avoid negative judgments. A mastery goal orientation is indicative of a growth mindset and is not dependent on a particular curriculum.

There are clear links between a mastery curriculum, mastery teaching and mastery goals but the suggestion of a mastery **level** in assessment, above what is expected, is problematic due to the conflict between this idea and the idea of a mastery curriculum being one where all children learn what is expected. The analysis of mastery mathematics in high-performing countries shows that the intention is to provide all children with full access to the curriculum, enabling them to achieve confidence and competence – ‘mastery’ – in mathematics’(NCETM, 2014). Bloom (Bloom, 1968) suggested that there were only two judgements to make in terms of assessment in a mastery curriculum, mastery or non-mastery, with non-mastery accompanied by detailed diagnosis and prescription of what is yet to be done before mastery is complete.

Whilst there are numerous descriptions of mastery in mathematics there are some common features of the different approaches. What the approaches have in common is an emphasis on success for all and that this can be achieved by developing conceptual understanding in mathematics, with a focus on mathematical structures. Most approaches advocate keeping the whole class together, not moving on until ideas are understood and promoting understanding through a variety of representations.

2. Mastery in Mathematics does not allow for any differentiation

There is a misunderstanding that when using a mastery approach, all students must be doing exactly the same work, with no differentiation for groups or individuals. This probably stems from ideas of keeping the whole class together working on the same topic. The National Curriculum programmes of study state:

... the expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. (DfE, 2013).

Additionally, Debbie Morgan, NCETM Director for Primary, states that two of the seven broad characteristics of teaching for mastery are ‘Teachers communicating their expectation that all pupils (except those with extreme special needs) will achieve’ and ‘Keeping the whole class together on the same material.’ (MathsHubs, 2015a)

These statements could imply that all children will receive the same provision and the expectation is that they will achieve equally, and it is likely that this is the origin of this second myth. However, the National Curriculum also suggests that pupils should be moved on only when they are conceptually ready and that pupils who have grasped concepts will be given ‘rich and sophisticated problems before any acceleration through new content.’ (DfE, 2013). This demonstrates that, within the curriculum, there is clear acknowledgement that not all children will develop a particular concept at the same time.

According to Charlie Stripp, Director of NCETM, one of the most common perceptions of differentiation in Primary School Mathematics in England involves provision of different tasks within a lesson according to perceived ability (Stripp, 2014). Compared to other countries, the UK has a slightly bigger gap between the highest and lowest attaining students in mathematics (PISA, 2014). Bloom suggests that dividing the class or year group according to perceived ability, becomes a fulfilling prophecy concerning students’ outcomes (Bloom, 1968). This concern is echoed by Dweck (Dweck, 2012) whose work suggests a link between mindset and attainment.

Mastery teaching on the NCETM website refers to ‘meeting the needs of all pupils without differentiation of *lesson content*’ (Stripp, 2015). This suggests that differentiation which requires teachers to produce different content for perceived ability groups, may be incompatible with mastery. This fits with a view that mastery learning is about efforts to reduce variation in student achievement and close achievement gaps (Guskey, 2009). The NCETM page goes on to clarify what differentiation might look like within a mastery curriculum. It is suggested that differentiation is not through subject content but through urgent intervention for those children who are not meeting objectives and, for those who have rapidly grasped the concept, enrichment rather than acceleration. Charlie Stripp, Director of NCETM, states that differentiation ‘*can be achieved by ‘same day intervention’ and ‘incorporating skilful questioning within whole class teaching’*’ (NCETM, 2015a). The theme of ‘same day’ or ‘rapid intervention’ also appears in books and articles eg (Drury, 2014), (NCETM, 2014).

Differentiation therefore *can* exist within a mastery approach and these appear to be the key strategies:

- Skilful questioning within lessons to promote conceptual understanding (Drury, 2014, Jones, 2014, Guskey, 2009)
- Identifying and rapidly acting on misconceptions which arise through same day intervention (Stripp, 2014, MathsHubs, 2015a) (ARK, 2015)
- Challenging, through rich and sophisticated problems, those pupils who grasp concepts rapidly, before any acceleration through new content. (NCETM, 2014)
- Use of concrete, pictorial and abstract representations sometimes linked to levels of conceptual development (Jones, 2014, Drury, 2014).

This last point is sometimes linked to differentiation and to a view that ‘less able’ children are more likely to need ‘concrete’ apparatus, while more able children can move straight to a pictorial or even abstract representation. In our view, skilful use by teachers of a variety of representations for pupils, enabling pupils themselves to represent mathematical in different ways, is part of effective teaching. Whereas, a rigid view of the suitability of particular representations for particular pupils is linked to a fixed ability self-theory (Dweck, 2000) and hence not conducive to pupil effort and to learning.

3. There is a special curriculum which is ‘The Mastery Curriculum’

This misunderstanding that there is only one Mathematics Mastery Curriculum is related to the first myth that there is only one definition of mastery. Furthermore a curriculum alone cannot provide a mastery approach. All valid approaches to mastery in mathematics involve guidance on teaching as well as on curriculum.

In the UK context there is a tension for the curriculum implicit in mastery approaches. In our experience the notion of curriculum coverage is deeply embedded in UK practice. However it is the teacher who covers the curriculum, not necessarily the learners. In a mastery approach by contrast it is the pupils who learn the topics in the curriculum. Many children and students currently do not learn all the topics taught to them; there are gaps. This implies that more time should be spent on those topics and there should be a reduction in the number concepts covered.

This leads to one of the key features in some descriptions of a mastery *curriculum* for mathematics - reducing the number of mathematical topics handled in class, taking longer over each one,

allowing all children to make sense of the mathematics (NCETM, 2015a); ‘more time on fewer topics’ (ARK, 2012). Bloom emphasised the importance of time for individuals to learn:

We believe that each student should be allowed the time he needs to learn a subject...The task...is to find ways of altering the time individual students need for learning as well as to find ways of providing whatever time is needed by each student. (Bloom, 1968)

There are different views on the compatibility of the current National Curriculum Programme of Study with mastery approaches. In internal discussions amongst NAMA members some have argued that the National Curriculum contains too much material. For example, Y6 contains nine domains, forty-nine statutory requirements and includes maths topics which were designated for Key Stage 3 in the previous version of the National Curriculum. Others have argued that the reduction in the number of concepts taught in key stage 1 will support mastery, if used with mastery approaches to teaching and learning. This secure foundation at key stage 1 could then enable concepts to be developed more easily as children get older. Furthermore, the stated aims of the current National Curriculum imply that mastery is intended. Hence the National Curriculum for Mathematics (2014) has been described as a ‘mastery curriculum’ (NCETM, 2014)

Some descriptions of a mastery *curriculum* for mathematics include a focus on small steps in the design of the curriculum:

Effective mastery curricula in mathematics are designed in relatively small carefully sequenced steps, which must each be mastered before pupils move to the next stage. Fundamental skills and knowledge are secured first. (NCETM, 2014)

However, this idea of small steps has been described as being in conflict with Bloom’s original ideas:

Another misinterpretation stems from... efforts focused only on low-level cognitive skills, attempted to break learning down into small segments, and insisted that students “master” each segment before being permitted to move on...Nowhere in Bloom’s writing, however, can this kind of narrowness and rigidity be found. In fact, Bloom emphasized quite the opposite. (Guskey, 2009)

One of the key features of Mathematics Mastery (ARK approach to mastery in mathematics) is ‘Always using objects and pictures before numbers and letters’ whilst ‘The teaching of critical thinking and problem solving skills is embedded’. It has been suggested that there is a tension between these two ideas on the Mathematics Mastery website (Blair, 2014):

If posing problems is as much at the heart of the mastery curriculum as the authors assert, then the “concrete, pictorial, abstract approach” to learning cannot be the straitjacket it is presented as. Alternatively, problem solving is tacked onto the end of the mastery sequence in much the same way as it is in many conventional classrooms. (ibid)

As indicated earlier, in our view a variety in representations of mathematical ideas is valuable, but inflexible use of different representations is not helpful. This applies to curriculum as well as to teaching. A curriculum that rigidly requires pupils to engage with concrete, pictorial and then abstract approaches to every concept is not necessary for mastery.

In a recent blog, Charlie Stripp, Director of NCETM makes the point that because of the inbuilt expectations, the new National Curriculum is a mastery curriculum. He makes the point that mastery relates to depth of understanding and that can only come from skilled teachers with a sound understanding of mathematics who have adequate time and resources available to teach children to the depth of understanding that is needed. This will not happen if the focus is on covering all the curriculum content – a legacy of the old curriculum and the pressure put on schools to secure ‘progress’ and to generate data for use by others.

In our view, mastery involves, (but does not consist of) a curriculum that is flexible, employs problem solving as an integral part, aims for fluency with understanding and supports the development of mathematical reasoning.

4. Mastery in Mathematics involves repetitive practice

This misunderstanding is based on the false idea that mastery relates only to mechanical procedures, rather than to concepts. Coupled with the idea that to master a procedure requires repeated practice at that procedure, this leads some teachers to believe that a mastery approach to mathematics requires repetitive practice with little variation in the items practiced for any particular procedure. This is not the case.

Notions of effective practice for students were given in The Cockcroft Report (par 239) (Cockcroft, 1982). Here a clear distinction is made between fluency that is built on understanding, and purely mechanical performance which does not in itself lead to long term retention or transfer of use to other contexts.

Para 239: ...we need to distinguish between ‘fluent’ performance and ‘mechanical’ performance. Fluent performance is based on understanding of the routine which is being carried out; mechanical performance is performance by rote in which the necessary understanding is not present. Although mechanical performance may be successful in the short term, any routine which is carried out in this way is much less likely either to be capable of use in other situations or to be retained in long term memory. (ibid)

No valid models of mastery in mathematics advocate the sort of practice that aims only at short term memory, rather than at conceptual understanding. Malcolm Swan has contributed to our discussion and we have this quote from an email he sent to us:

If practice is just repeating the same procedure with different numbers, chosen randomly, then it has no purpose. Some appear to think that such practice is like training a muscle, where repeated exercise builds up some kind of inner mental strength and speed. In fact it usually results in boredom. Variation theory tells us that by systematically changing significant aspects of a task, keeping the rest fixed, we can focus the students’ attention on those aspects and conceptual change can result. But the emphasis in making such variations is not to develop speed but to develop an awareness of pattern, leading to conjecture, generalisation, explanation and deeper understanding.

This systematic changing of aspects of a task is key to designing effective practice activities for students. This point is made well in a conference paper by Anne Watson and John Mason.

Our conclusions after three years of work in a range of natural settings are that control of dimensions of variation and ranges of change is a powerful design strategy for producing

exercises that encourage learners to engage with mathematical structure, to generalize and to conceptualize even when doing apparently mundane questions. (Watson and Mason, 2006)

Furthermore in China, procedural variation is used to promote deep understanding of mathematics (Lai and Murray, 2012). So the types of practice promoted by mastery in mathematics include use of a concept or procedure in a *variety* of contexts.

Using an analogy with musical practice, mathematical études consist of mathematical tasks which embed the practice of essential techniques within a richer, exploratory and investigative context (Foster, 2013). Such mathematical tasks are focused on developing genuine fluency in skills, based on conceptual understanding and through extensive opportunities for rehearsal alongside more thoughtful and mathematically creative activity.

In summary, there is a danger that superficial repetitive practice becomes simply a mechanical exercise, quickly lost in memory and difficult to apply in different contexts. However, where practice is developed in an organised way to create focus on a higher level of understanding then such practice becomes particularly useful.

5. Mastery in Mathematics means you have to use particular text books

In autumn 2014, the Minister of State for School Reform Nick Gibb MP, responding to a paper published to coincide with a conference held by the Publishers Association, outlined his hope that the paper would

“... lead to the renaissance of intellectually demanding and knowledge-rich textbooks in England’s schools”. (DfE and Gibb, 2014)

The paper was written by Tim Oates, who chaired the 2010 National Curriculum review; it looks at the highest performing jurisdictions around the world and analyses a range of features including the use of textbooks (Oates, 2014). The author highlights an opposition to textbooks among many educationalists, as well as a failure of the market in England. He says textbooks have been largely abandoned in favour of the use of worksheets and ‘*myopic*’ exam-based books, in stark contrast to places such as Singapore, Finland and Shanghai, where high-quality textbooks are a key part of the classroom, supporting learners and teachers alike.

Summing up the situation, he says:

*We may not have been conscious of the movement in England away from the wide use of high quality textbooks, but it **has** happened. ... We’ve ... failed to notice the emergence, in other nations, of extremely well-theorised, well-designed, and carefully implemented textbooks.’* (Oates, 2014) (Page 4).

Tim Oates concludes by calling for ‘*self-searching criticism of the status-quo in England*’ and for a ‘*concerted effort by publishers, the state, researchers and educationalists*’ (page 20) to align more with emerging international standards of excellence on textbooks. He warns that England has been overtaken by the highest-performing education systems, partly because they value textbooks so highly.

It is in this context that a policy decision was made to produce new text books for England. 'Textbooks and Professional Development' is one of five national collaborative projects that the MathsHubs are engaged in. This trial draws project draws on evidence from the Singapore curriculum, adapting ideas from their text books to the mathematic programme of study for England. It involves the development and trialling by two UK publishers of primary mathematics schemes (*Maths – No Problem* and *Inspire Maths*) starting in Y1 to help participating teachers and schools develop and embed a mastery approach to maths teaching (MathsHubs, 2015b). Professional development provided to participating teachers is a key feature of this project (NCETM, 2015b). Our discussions within NAMA revealed that some schools, more remote from the MathsHubs, seem to have developed a misunderstanding that it is only the use of these particular text books which constitutes implementing mastery in mathematics.

The use of any particular textbook per se does not guarantee a mastery approach. Furthermore there is a risk that textbooks can limit expectations and aspirations. There may be a notion that they cannot be changed, or if there are changes these cannot be shared beyond the classroom or school (Drury, 2015). Indeed there may be an assumption that the textbook provides all the answers, hence limiting the freedom of choice the teacher feels they have when selecting examples to use with children. There may be a reluctance to drop examples in the text book that the teacher feels are inappropriate for their children, in the belief that only by successful completion of all examples in the text book will there be mastery of the topic.

Textbooks should give learners good examples that deepen understanding and they should support teachers in developing this deeper understanding for learners of key mathematical concepts. It is often the case that the best teaching observed in England has been evident when the teacher has chosen the learning tasks from a wide range that is available, ensured that a problem solving approach is integrated (Ofsted, 2011) and has regularly checked learners' understanding and knowledge development throughout the learning experience (Ofsted, 2013). The ability to solve problems in a range of contexts, to reason mathematically and to be fluent and confident with number and the number system is embodied in the aims of the National Curriculum and is at the heart of 'mastery'.

In conclusion, good textbooks are a tool which can support teachers and learners in developing mastery in mathematics but cannot be used to replace a range of effective requirements which are part of good teaching. For example, they cannot be a substitute for effective questioning and the requirement that learners reason and justify their findings and solutions. Good textbooks should support and extend teachers' capacity and skills in questioning and other aspects of high quality teaching that enable all learners to be confident, fluent and successful in mathematics. High quality textbooks have a role to play in developing learners' mastery in mathematics as long as they are well-designed and well-theorised to support teachers in developing effective practice.

Conclusion

In our work with schools we want to do our best to help them adopt approaches that are most likely to lead to success for pupils. Much of our work is directly concerned with mathematics teacher development. We hope that by having a better understanding ourselves of the various ways that the term 'mastery' is being used we will be better placed to support teachers. It is hoped also that this

document, by alerting the reader to possible misunderstandings about mastery in mathematics, will help advisers in their work with schools.

Our general view is that a focus on the common positive features of mastery will be helpful to schools. It is worth remembering that within high performing countries, where mastery approaches could be said to be prevalent, disadvantaged pupils do as well as their peers (PISA, 2014). However, we would caution against assuming a simple causal relationship between the systems in high performing jurisdictions and their outcomes. Outcomes are the result of more than the education system; in particular, parental attitudes and wider cultural values and beliefs are implicated.

There is much sound advice to be found about mastery in mathematics and much to support a view that mastery approaches will lead to good outcomes. For example, the Mathematics Mastery programme (ARK) claim that mastery is

... a highly effective approach to the teaching and learning of mathematics for all and could have particular benefits for pupils who tend to fall behind in the current system (ARK, 2015).

Furthermore Bloom suggests that a mastery approach could support a narrowing of the 'attainment gap' (Bloom, 1971a). Providing success for pupils in England who are disadvantaged currently is one of the key purposes of the support that NAMA members provide for schools, so we welcome any approach that helps with this aim.

To summarise some points from the body of this document, mastery in mathematics involves a commitment that all children can and will achieve; teaching less but in more depth and effective questioning to root out misconceptions and rapidly address these. We would argue that these practices are evidence of good teaching and are not solely attached to one approach.

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